Effects and Estimation Techniques of Symbol Time Offset and Carrier Frequency Offset in OFDM System: Simulation and Analysis

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Abstract – Orthogonal frequency division multiplexing system is multichannel system that provides high rate and high quality wireless communication services, but system performance is degraded by two factors, one is the symbol timing offset (STO) and other is the carrier frequency offset (CFO) caused by Doppler frequency shift. Several approaches have been proposed to estimate time and frequency offset. Most frequency and timing estimation methods use the periodic nature of the time domain signal by using a cyclic prefix or by designing the training symbol having repeated parts. In this paper, the effects of STO and CFO in OFDM system have been studied. Also, the estimation techniques to handle the STO and CFO problems in OFDM system by using cyclic prefix and training symbol has been analyzed.

Keywords- Carrier Frequency Offset, Inter Channel Interference, Inter Symbol Interference, Symbol Time Offset, OFDM

I. INTRODUCTION

Multi path fading and bandwidth efficiency are two main difficult challenges in the future wireless communication system. OFDM system reduces the multi path fading effect by using a parallel collection of frequency flat sub channels and it has been found in broad applications such as WLAN IEEE 802.11a/n, Digital Audio Broadcasting (DAB), Digital Terrestrial Video Broadcasting (DVB-T), IEEE 802.16a/e, Ultra-wideband (UWB). The OFDM system is multi carrier system that carries the message data on orthogonal subcarriers for parallel transmission, combating the distortion caused by the frequency selective channel and the inter symbol interference in the multi-path fading channel. The carrier spacing is carefully selected so that each sub carrier is orthogonal to the other sub carriers. The OFDM system advantage can be useful only when the orthogonality is maintained. As the sub carriers are orthogonal, the spectrum of each carrier has a null at the center frequency of each of the other carriers in the system. This results in no interference between the carriers, allowing them to be spaced as close as theoretically possible. There are two types of distortion associated with the carrier signal [1], that is the symbol time offset (STO) and carrier frequency offset (CFO). The sensitivity to timing and carrier offset errors is higher in OFDM systems than in single carrier systems. The symbol timing offset is modeled as a delay in the channel impulse response and carrier frequency offset, which is due to a difference in the local oscillators in the transmitter and receiver, gives rise to a shift in the frequency domain. This carrier frequency offset causes loss of orthogonality between sub-carriers and then the signals transmitted on each carrier are not independent of each other. The symbol times offset is represented as phase rotation and inter symbol interference (ISI). Carrier frequency offset is represented as phase rotation and inter channel interference (ICI) [2]. The received baseband signal under the presence of CFO $\epsilon$ and STO $\delta$ can be expressed as

$$y_i[n] = IDFT(Y_i[k]) = IDFT(H_i[k]X_i[k] + Z_i[k])$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} H_i[k]X_i[k] e^{j2\pi(k+\epsilon)(n+\delta)/N} + z_i[n]$$

(1)

(2)
Where $\varepsilon$ and $\delta$ denotes the normalized CFO and STO and $X_i[k], Y_i[k], H_i[k], Z_i[k]$ are the $k$th subcarrier frequency components of $i$th transmitted symbol, received symbol, channel frequency response and noise in frequency domain.

This paper is organized as follows; Section II describes the effects of STO and Section III describes the effects of CFO on OFDM system performance. The STO and CFO estimation techniques, performance evaluation, and simulation results are provided in Section IV and Section V. Finally, conclusions are provided in Section VI.

II. ANALYSIS OF SYMBOL TIMING OFFSET EFFECTS IN OFDM SYSTEM

In OFDM system, the modulation and demodulation requires the IFFT and FFT functions at the transmitter and receiver, respectively. In order to take the $N$-point FFT in the receiver, the exact samples of the transmitted signal are needed for the OFDM symbol duration. In other words, a symbol timing synchronization must be performed to detect the starting point of each OFDM symbol, which facilitates obtaining the exact samples. The STO of $\delta$ samples affects the received symbols in the time and frequency domain. The STO of $\delta$ in the time domain incurs the phase offset of $2\pi \delta / N$ in the frequency domain, which is proportional to the subcarrier index $k$ and $\delta$. The effect of STO depends upon the location of the estimated starting point of OFDM symbol.

Considering the sample indexes of a perfectly synchronized OFDM symbol be $\{0, \ldots, N-1\}$ and the timing offset be $\delta$, and the maximum channel delay spread be $\tau_{max}$. When $\delta \in \{-N_{\tau}, \ldots, -1, 0, 1, \ldots, N_{\tau}\}$, the orthogonality among the subcarriers will not be destroyed by the resulting inter symbol interference (ISI) and additional inter carrier interference (ICI) given as

$$Y_i[k] = \frac{1}{N} \sum_{n=0}^{N-1} x_i[n + \delta] e^{-j2\pi nk / N}$$

$$= X_i[K] e^{j2\pi \delta / N} \quad (3)$$

There exist a phase offset that is proportional to the STO $\delta$ and subcarrier index $k$. If the timing estimate is outside the above range, the orthogonality among the subcarriers will be destroyed by the resulting inter symbol interference (ISI) and additional inter carrier interference (ICI) given as

$$Y_i[k] = \frac{N - \delta}{N} X_i[p] e^{j2\pi(p-k)n / N} + \sum_{p=0, p \neq k}^{N-1} X_i[p] e^{j2\pi(p-k)n / N} + \frac{1}{N} \sum_{p=0}^{N-1} X_i[p+1] e^{j2\pi(p-k)n / N} (4)$$

$$+ \frac{1}{N} \sum_{n=0}^{N-1} X_i[p+1] e^{j2\pi p(N-\delta) / N} \sum_{n=N-\delta}^{N-1} e^{j2\pi(p-k)n / N} \quad (5)$$

Where

$$\sum_{n=0}^{N-1-\delta} e^{j2\pi(p-k)n / N} = e^{j2\pi(p-k)N-\delta} \frac{\sin \left( \frac{(N-\delta)\pi(k-p)}{N} \right)}{\sin \left( \frac{\pi(k-p)}{N} \right)}$$

$$= \begin{cases} 
(N-\delta) & \text{for } p = k \\
\text{Nonzero} & \text{for } p \neq k 
\end{cases} \quad (6)$$

The second term in the Equation (5) corresponds to ICI, which implies that the orthogonality has been destroyed. Furthermore, it is also clear from the third term in Equation (5) that the received signal involves the ISI (from the next OFDM symbol $X_i[p+1]$).
III. ANALYSIS OF CARRIER FREQUENCY OFFSET EFFECTS IN OFDM SYSTEM

The carrier frequency offset (CFO) is caused by Doppler frequency shift $f_d$. Let $f_c$ and $f'_c$ denote the transmitter and receiver carrier frequencies, respectively. The difference between them is denoted by $f_{offset}$ ($f_{offset} = f_c - f'_c$). The normalized CFO, $\varepsilon$ is defined as a ratio of the CFO to subcarrier spacing $\Delta f$, given as

$$\varepsilon = \frac{f_{offset}}{\Delta f}$$  \hspace{1cm} (7)

For time domain signal $x[n]$, a CFO of $\varepsilon$ causes a phase shift of $2\pi \varepsilon n$ which is proportional to the CFO and time index $n$. It is equivalent to a frequency shift of $-\varepsilon$ on the frequency-domain signal $X[k]$. The time domain received signal can be written as

$$y_i[n] = \frac{1}{N} \sum_{k=0}^{N-1} H[k]X_i[k]e^{j2\pi(k+\varepsilon)n/N} + z_i[n] \hspace{1cm} (8)$$

The normalized CFO can be divided into two parts: integer CFO (IFO) $\varepsilon_i$ and fractional CFO (FFO) $\varepsilon_f$ that is $\varepsilon = \varepsilon_i + \varepsilon_f$. Due to the IFO, the transmitted signal with $X[k]$ is cyclically shifted by $\varepsilon_i$ in the receiver, and thus the received signal $X[k - \varepsilon_i]$ in the $k$th subcarrier. Unless the cyclic shift is compensated, it will incur a significant degradation in the BER performance. The orthogonality among the subcarrier frequency components is not destroyed and thus, ICI does not occur. The frequency domain received signal with an FFO of $\varepsilon_f$ can be written as [4]

$$Y_i[k] = \frac{\sin(\pi\varepsilon_f)}{N \sin(\pi\varepsilon_f/N)} e^{j\pi\varepsilon_f(N-1)/N} H[k]X_i[k] + I_i[k] + Z_i[k] \hspace{1cm} (9)$$

Where

$$I_i[k] = e^{j\pi\varepsilon_f(N-1)/N} \sum_{m=m_k}^{N-1} \frac{\sin\left(\pi(m-k+\varepsilon_f)\right)}{N \sin(\pi(m-k+\varepsilon_f)/N)} H[m]X_i[m]e^{j\pi(m-k)(N-1)/N} \hspace{1cm} (10)$$

The first term of Equation (9) represents the amplitude and phase distortion of the $k$th subcarrier frequency component due to FFO. Meanwhile, $I_i[k]$ in Equation (9) represents the ICI from other subcarriers into $k$th subcarrier frequency component, which implies that the orthogonality among subcarrier frequency components is not maintained any longer due to the FFO.

IV. ESTIMATION OF SYMBOL TIMING OFFSET

An STO causes phase distortion as well as ISI in OFDM systems. To improve its performance, the starting point of OFDM symbols must be accurately determined by estimating the STO with a synchronization technique at the receiver. The STO estimation can be implemented either in the time or in frequency domain. Let $N_C$ be the number of samples of cyclic prefix (CP) over $T_C$ seconds and $N_{sub}$ be the number of samples of effective data over $T_{sub}$ seconds. In the time domain, STO can be estimated by using CP or training symbols.

A. STO Estimation Techniques Using Cyclic Prefix (CP)

The cyclic prefix is a replica of the data part in the OFDM symbol. The similarities between the CP and the corresponding data part can be used for STO estimation. The CP part and the corresponding data part are taken as two sliding windows $W_1$ and $W_2$ that are spaced by $N_{sub}$ samples. These windows can slide to find the similarity between the samples within $W_1$ and $W_2$. The similarity between two blocks of $N_C$ samples in $W_1$ and $W_2$ is maximized when CP of an OFDM symbol falls into the first sliding window. Since the similarity between two
blocks in $W_1$ and $W_2$ is maximized when the difference between them is minimized, the STO can be found by searching the point where the difference between two blocks of $N_g$ samples within these two sliding windows is minimized [5].

$$\hat{\delta} = \arg \min_\delta \left\{ \sum_{i=0}^{N_g-1+\delta} |y_i[n+i] - y_i[n+N+i]| \right\}$$ (11)

The system performance also can be degraded by CFO. The other STO estimation technique, is to minimize the squared difference between $N_g$ sample block (seized in window $W_1$) and the conjugate of another $N_g$ sample block (seized in window $W_2$) [6], given as

$$\hat{\delta} = \arg \min_\delta \left\{ \sum_{i=0}^{N_g-1+\delta} (|y_i[n+i] - y_i^*[n+N+i]|)^2 \right\}$$ (12)

The correlation between those two blocks in sliding windows can also be used for STO estimation. For this the maximum-likelihood estimation scheme can be applied.

$$\hat{\delta} = \arg \max_\delta \left\{ \sum_{i=0}^{N_g-1+\delta} |y_i[n+i]y_i^*[n+N+i]| \right\}$$ (13)

This equation is also affected by CFO. To combat this ML can be used that maximizes the log likelihood function, given as

$$\hat{\delta}_{ML} = \arg \max_\delta \left\{ \sum_{i=0}^{N_g-1+\delta} 2(1-\rho)\text{Re}[y_i[n+i]y_i^*[n+N+i]] - \rho \sum_{i=0}^{N_g-1+\delta} |y_i[n+i] - y_i[n+N+i]| \right\}$$ (14)

where $\rho = SNR/(SNR + 1)$ [7]. The estimation of both STO and CFO can be done at same time using ML technique [8]. The STO is estimated as

$$\hat{\delta}_{ML} = \arg \max_\delta \left\{ |\gamma[\delta]| - \rho \Phi[\delta] \right\}$$ (15)

where

$$\gamma[m] = \sum_{n=m}^{m+i-1} y_i[n]y_i^*[n+N]$$ (16)

and

$$\Phi[m] = \frac{1}{2} \sum_{n=m}^{m+i-1} (|y_i[n]|^2 + |y_i[n+N]|^2)$$ (17)

$L$ denotes the actual number of samples used for averaging in windows. The Figure 1 shows the STO estimation using CP. It employs the maximum correlation based technique by equation (13) which estimates STO by maximizing the correlation between CP and corresponding part of OFDM symbol and the minimum difference based technique by equation (12) estimating by minimizing the difference between CP and corresponding part of OFDM symbol.
B. STO Estimation Techniques Using Training Symbol

Training symbols can be transmitted to be used for symbol synchronization in the receiver. Two identical OFDM training symbols or a single OFDM symbol with a repetitive structure can be used. The repetitive pattern in the time domain can be generated by inserting 0s between subcarriers. Once the transmitter sends the repeated training signals over two blocks within the OFDM symbol, the receiver attempts to find the CFO by maximizing the similarity between these two blocks of samples received within two sliding windows. The similarity between two sample blocks can be computed by an autocorrelation property of the repeated training signal. As STO can be estimated by minimizing the squared difference between two blocks of samples received in $w_1$ and $w_2$ [9, 10], such that

$$\delta = \arg \min_\delta \left\{ \sum_{i=\delta}^{N-1+\delta} |y_i[n + i] - y_i^*[n + \frac{N}{2} + i]|^2 \right\} \quad (18)$$

By maximizing the likelihood function [9]

$$\hat{\delta} = \arg \min_\delta \left\{ \frac{1}{\sum_{i=\delta}^{N-1+\delta}} |y_i[n + i]y_i^*[n + \frac{N}{2} + i]|^2 \right\} \quad (19)$$

These techniques have advantages of estimating STO without being effected by CFO.

C. Frequency Domain Estimation Techniques for STO

The STO results in phase rotation in the received signal; the phase rotation is proportional to subcarrier frequency. The STO can be estimated by the phase difference between adjacent subcarrier components of the received signal in the frequency domain. When $X_i[k] = X_i[k - 1]$ and $H_i[k] \approx H_i[k - 1]$ for all $k$, then $Y_i[k]Y_i^*[k - 1] \approx |X_i[k]|^2e^{j2\pi \delta}$. The STO can be estimated as

$$\hat{\delta} = \frac{N}{2\pi} \arg \left( \sum_{k=1}^{N-1} Y_i[k]Y_i^*[k - 1] \right) \quad (20)$$
An STO can be estimated from channel impulse response which is obtained by multiplying the received symbol (with STO) by the conjugated training symbol \( X_i^*[k] \).

\[
\delta = \arg\max_n (y_i^X[n]) \quad (21)
\]

where

\[
y_i^X[n] = \text{IFFT} \left\{ \frac{1}{N} Y_i[k] e^{\frac{j2\pi k\delta}{N}} X_i^*[k] \right\}
\]

\[
= h_i[n + \delta] \quad (22)
\]

The power of training symbol \( X_i^*[k] \) is equal to one \((X_i^*[k]X_i^*[k] = |X_i^*[k]|^2 = 1)\). Figure 2 shows the results of frequency domain STO estimation using channel impulse response.

![STO estimation results](image)

V. ESTIMATION OF CARRIER FREQUENCY OFFSET

By estimating the frequency offset at the receiver, the loss in performance due to a frequency mismatch of the received signal and the receive oscillator can be significantly reduced. CFO estimation can also be performed either in the time or the frequency domain.

A. CFO Estimation Techniques Using Cyclic Prefix (CP)

A CFO results in phase rotation of \( 2\pi \delta N / N \) in the received signal. The phase difference between CP and the corresponding part of an OFDM symbol (spaced \( N \) samples apart) caused by CFO is \( \frac{2\pi \delta N}{N} = 2\pi \delta \). The CFO can be estimated from the product of CP and the corresponding real part of an OFDM symbol as

\[
\hat{\delta} = \frac{1}{2\pi} \arg \left\{ \frac{1}{N} \sum_{n=-N_g}^{N_g-1} y_i^*[n] y_i[n+N] \right\}
\]

\[
= \frac{1}{2\pi} \arg \left\{ \sum_{n=-N_g}^{N_g-1} y_i^*[n] y_i[n+N] \right\} \quad (24)
\]
CFO estimation lies in the range of \((-\pi/2, +\pi/2)\) so that \(|\varepsilon| < 0.5\). The term \(y_i^*[n]y_i[n + N]\) becomes real only when there is no frequency offset. It becomes imaginary as long as the CFO exists. In fact, the imaginary part of \(y_i^*[n]y_i[n + N]\) can be used for CFO estimation \([11]\). In this case, the estimation error is defined as

\[
e_\varepsilon = \frac{1}{L} \sum_{n=1}^{L} \text{Im}(y_i^*[n]y_i[n + N])
\]

where \(L\) is the no. of samples used for averaging. The expectation of error function can be given as

\[
E(e_\varepsilon) = \frac{\sigma_d^2}{N} \sin \left( \frac{2\pi \varepsilon}{N} \right) \sum_{k=0}^{L} |H_k|^2 \approx K_\varepsilon
\]

\(\sigma_d^2\) is transmitted signal power, \(H_k\) is channel impulse response of \(k\)th subcarrier.

### B. CFO Estimation Techniques Using Training Symbol

The CFO estimation technique using CP can estimate the CFO only within the range \(|\varepsilon| < 0.5\) but the can be large at the initial synchronization stage. The range of CFO estimation can be increased by reducing the distance between two blocks of samples for correlation. This is made possible by using training symbols that are repetitive with some shorter period. The ratio of the OFDM symbol length to the length of a repetitive pattern is denoted as \(D\). A transmitter send the training symbols with \(D\) repetitive patterns in the time domain, which can be generated by taking the IFFT of a comb-type signal in the frequency domain given as

\[
x_i[k] = \begin{cases} 
A_m, & \text{if } k = D, i = 0, 1, \ldots, \left\lfloor \frac{N}{D} \right\rfloor - 1 \\
0, & \text{otherwise}
\end{cases}
\]

where \(A_m\) is an M-ary symbol and \(N/D\) is an integer. As \(x_i[n]\) and \(x_i[n + N/D]\) are identical, a CFO estimation can be as

\[
\hat{\varepsilon} = \frac{D}{2\pi \arg \left( \sum_{m=0}^{N-1} y_i^*[n]y_i[n + N/D] \right)}
\]

The CFO estimation range is \((|\varepsilon| \leq D/2)\). This increase in estimation range is obtained at the sacrifice of MSE (mean square error) performance. As the estimation range of CFO increases, the MSE performance becomes worse. By taking the average of the estimates with the repetitive patterns of the shorter period as

\[
\hat{\varepsilon} = \frac{D}{2\pi \arg \left( \sum_{m=0}^{N-1} \sum_{n=0}^{D-1} y_i^*[n + mN/D]y_i[n + (m+1)N/D] \right)}
\]

### C. Frequency Domain Estimation Techniques for CFO

When two same training symbols are transmitted consecutively, then the signals with CFO of \(\varepsilon\) are related to each other as

\[
y_2[n] = y_1[n]e^{j2\pi\varepsilon N/n}
\]

The CFO can be estimated as

\[
\hat{\varepsilon} = \frac{1}{2\pi \tan^{-1} \left( \sum_{k=0}^{N-1} \text{Im}[Y_1^*[K]Y_2[k]] / \sum_{k=0}^{N-1} \text{Re}[Y_1^*[K]Y_2[k]] \right)}
\]

Generally the synchronization process is divided into two phases acquisition and tracking. In the acquisition phase, a rough estimate of the frequency offset is obtained and corrected. The residual small deviations are then
corrected in the tracking mode. Furthermore, conditions are not static in a real system, this causes small deviations in frequency offset that the tracking stage should estimate and correct. This approach is given by Moose. The range of CFO estimation is \[ |\epsilon| \leq \frac{\pi}{2T} = 1/2. \]

The other technique is proposed by Classen [12], in which the pilot tones can be inserted in the frequency domain and transmitted in every OFDM symbol for CFO tracking. The CFO is estimated by:

\[
\hat{\epsilon}_{acq} = \frac{1}{2\pi T_{sym}} \max_j \left\{ \frac{1}{L-1} \sum_{i=0}^{L-1} Y_{i+j}[p[j], \epsilon] X_{i+j}^* [p[j], \epsilon] \right\}
\]

(31)

L, \( p[j] \) and \( X_{i+j}[p[j]] \) denotes the no. of pilot tones, the location of the jth pilot tone, and the pilot tone located at \( p[j] \) in the frequency domain at the lth symbol period, respectively.

Figure 3 shows the CFO estimation by equation (29) using the phase difference between CP and corresponding part of OFDM symbol, equation (30) using the phase difference between two repetitive preambles, equation (31) using the phase difference between pilot tones in two OFDM symbols.

It is clear from that the mean squared CFO estimation errors decrease as the SNR of received signal increases. The performance of these techniques depends upon the no. of samples in CP, no. of samples in preamble and no. of pilot tones.

VI. CONCLUSION

This paper presents the performance of the timing and frequency offset estimators by computer simulation. The simulation results for CP based STO estimation is obtained using maximum correlation and minimum difference based techniques. Similarly, CFO estimation results are obtained using CP based estimation and estimation technique proposed by Moose and Classen.

VII. REFERENCES


