Mathematical Model Estimation for Falling Ball in Water Based on the Capture Images Using SONY Camera

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Abstract- In this research possible location measurement, displacement, velocity and acceleration of a falling balls in the water with different masses and volumes, using waterproof camera type(SONY),this by build a computer algorithms to analysis the motion in image planes for the successive frames and determine the falling object location, speed and Acceleration . After that using fitting processing to estimate the best fitting model for the motion parameters.then comparing between the real motion parameter deter as a function of times and the estimated values from the estimated model. Where have been found high a agreement between them

I. Introduction

Underwater image analysis and enhancement techniques offers a method for improving the object identification in underwater environment. A lot of research began for the improve the underwater image quality. But limited work has been done in the field of underwater images. Because they are in the form of lack of clarity in the environment under the water due to the conditions of poor visibility effects, such as the absorption of light, reflection of light, and the bending of light, medium density and the light scattering. These are important factors which cause the deterioration of underwater images[1]. In the past, research in image processing is limited mainly to normal images with the exception of few approaches that was applied to underwater images, [2,3]. Last few years, an increasing interest in marine research has encouraged researchers from various disciplines to explore the mysterious underwater world. A large amount of literature is available on image processing, ‘event detection’, ‘detection and tracking objects’, ‘feature detection’ and so forth.[4]

Case of a free falling (or rising) ball with velocity in an unlimited environment is related to many of the practical problems. Multiple applications in the chemical & metal processes, in sediment transport and precipitation in channels and pipes required the specification of the drag coefficient and the settling velocity of spherical bubbles, drops or particles,[5]. Many researches have been conducted to study the object motion in fluids, Some of these studies are as follow:

D.D.Joseph,et.al.[6] in 1994 they focuses Aggregation and dispersion of balls falling in viscoelastic liquids, They found maximum various between Newtonian and viscoelastic fluid, with repulsion between nearby bodies in the Newtonian case and attraction in the viscoelastic case .

Also A.A.Johnson, T.E. Tezduyar[7] in 1996 they have applied 3D parallel finite element simulation strategies for liquid particle interaction to study multiple balls falling in a fluid filled tube. They have been that the multiple balls tend to form stable geometric arrangements and the velocity that the balls falls is affected by the number of balls.

M. Brizard ,et.al.[8] in 2005 they designed experimental bench of viscosity measurement in order to follow the trajectory of the ball, to obtain the velocity evolution through the fall.

While Abbas H. Sulaymon,et.al. [9] in 2011 they have been conducted on the hydrodynamic interaction between two balls in Newtonian and non Newtonian liquid. Glycerin and polyacrylamide solution were used in which the interact balls were suspended. Comparison of the results shows that the ratio of drag coefficient in power-law liquid is exponentially dependent on the separation distance and is closely linked to the particle Reynolds number.

 Açmae El Yacoubi ,et.al.[10] in 2012 they Motivated by their desire for understanding collective behaviour and self-organization resulting from hydrodynamic interactions, they investigate the two-dimensional dynamics of horizontal arrays of settle cylinders at intermediate Reynolds numbers. To simulate these dynamics, they develop a direct numerical simulation based on immersed interface style.

M.D. Mikhailov,et.al.[6] in 2013 they Provided an accurate model for the drag coefficient (C_d) of a falling ball in terms of a non-linear rational fractional transform of the series of Goldstein to Oseen’s equation. Improved the coefficients of the six polynomial terms by a direct fit to the experimental data of others researchers.

II. Liquid Viscosity

The viscosity of a liquid is a measure of the internal friction opposing the deformation or flow of the liquid,[11]. In liquid dynamics, drag (sometimes called air resistance, a type of friction, or liquid resistance, another type of friction or liquid friction) refers to forces acting opposite to the relative motion of any object moving with respect to a surrounding liquid. This
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can exist between two liquid layers (or surfaces) or a liquid and a solid surface. Unlike other resistant forces, such as dry friction, which are nearly independent of speed, drag forces depend on speed,[12,13]. Drag force is proportional to the speed for a laminar flow and for a squared speed for a turbulent flow. Though the ultimate reason a drag is viscous friction, the turbulent drag is independent of viscosity,[13].

Wave drag occurs when a solid object is moving through a liquid at or near the speed of sound in that liquid—or in case there is a freely-moving liquid surface with surface waves radiating from the object, e.g. from a ship,[13]. Drag depends on the properties of the liquid and on the size, shape, and object velocity. The way to express this is means of the drag equation,[9]:

\[ F_d = \frac{1}{2} \rho_{\text{liquid}} v^2 c_d A \]  

(1)

Where, \( F_d \) is the drag force, \( \rho_{\text{liquid}} \) is the density of the liquid, \( v \) is the velocity of the object relative to the liquid, \( A \) is the cross-sectional area, and \( c_d \) is the drag coefficient—a dimensionless number. The drag coefficient depends on the object shape and on the Reynolds number (\( R_e \), dimensionless parameter that represents the ratio of viscous to inertial forces in a fluid),[14]:

\[ R_e = \frac{v D}{\mu'} \]  

(2)

Where, \( D \) is some characteristic diameter or linear dimension and \( \mu' \) is the kinematic viscosity of the liquid (equal to the viscosity \( \mu \) divided by the density). At low Reynolds number, the drag coefficient is asymptotically proportional to the inverse of the Reynolds number, means that the drag is proportional to the velocity. At high Reynolds number, the drag coefficient is more or less constant. Figure (1) shows how the drag coefficient varies with Reynolds number for the case of a ball,[13].

![Figure 1. The drag coefficient \( C_d \) as a function of Reynolds number \( R_e \) for a sphere, as obtained from laboratory experiments. The solid line is for a sphere with a smooth surface, while the dashed line is for the case of a rough surface.[13].](image)

### III. Ball Motion In Liquids

The force \( F_d \) required to drag a ball of radius \( r \) at velocity \( v \) through a liquid of viscosity \( \mu \) can be calculated from,[15,16]:

\[ F_d = 6\pi r \mu v \]  

(3)

This equation is defined as Stokes' Law and is valid only for laminar flow, where the flow of the liquid can be treated as consisting of layers. Each layer having a well defined velocity. Laminar flow around an airfoil is illustrated in Fig.(2a). Turbulent (non-laminar) flow is illustrated in Fig.(2b).
Figure 2: (a) Photographs showing (left) laminar flow, (b) (right) turbulent flow of a liquid around a wing-shaped airfoil.[16]

Figure 3 shows a diagram of the entire system (ball falling through a column of liquid). A free body diagram (FBD) from the sphere is the dashed cross-section that has been removed and exploded in the left part of this figure. [14]

![Free-body diagram of a ball in a quiescent liquid](image)

The FBD in this figure lists three forces acting on the ball: $F_b$, $F_d$, and $mg$. The first two forces arise from the buoyancy effect of displacing the liquid in question, and from the viscous drag of the liquid on the ball, respectively. Both forces act upwards -- buoyancy tending to 'float' the ball ($F_b$) and the drag force ($F_d$) resisting the acceleration of gravity. The only force acting down-wards is the body force resulting from gravitational attraction ($mg$). By summing forces in the vertical direction can be write the following equation,[14]

$$F_b + F_d = mg \quad (4)$$

The buoyancy force ($F_b$) is simply the weight of displaced liquid, the volume of a ball ($V_{ball}$) is written as,[14]:

$$V_{ball} = \frac{4}{3} \pi r^3 \quad (5)$$

Combining this volume with the mass density of the liquid, $\rho_{liquid}$, we can now write the buoyancy force as the product ,[14]:

$$F_b = m_{dis} g = \frac{4}{3} \pi r^3 \rho_{liquid} g \quad (6)$$

where $g$ is the gravitational acceleration and $r$ is the radius of the ball. Combining all of the previous relationships that describe the forces acting on the ball in a liquid we can write the following expression,[14]

$$\frac{4}{3} \pi r^3 \rho_{liquid} g + 6 \pi r \mu v = mg \quad (7)$$

A ball of density $\rho_{ball}$ falling in a stationary liquid of density $\rho_{liquid}$ feels a gravitational force($F_g$) due to the difference of the weight of the ball and the buoyancy on the ball, (both caused by gravity) given by,[14]:

$$F_g = \frac{4}{3} \pi r^3 g (\rho_{ball} - \rho_{liquid}) \quad (8)$$

Where, $g$ is the acceleration of gravity. when force balance: $F_d = F_g$ and solving for the velocity $v$ gives the terminal velocity $v_t$ note that since buoyant force increases as $r^3$ and Stokes drag increases as $r$, the terminal velocity $v_t$ increases
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as \( r^2 \) and thus varies greatly with particle size. If the particle is falling in the viscous liquid under its own weight due to gravity, then a terminal velocity \( (v_t) \), is reached when this frictional force combined with the buoyant force exactly balances the gravitational force. The resulting terminal velocity \( (v_t) \) is,[14]

\[
v_t = \frac{2gr^2}{9\mu} (\rho_{ball} - \rho_{liquid})
\]  

(9)

Experimentally determine the viscosity of a fluid since all of the remaining quantities in Eq.(9) can be measured easily.

where \( v_t \) is the flow terminal velocity (m/s) (vertically downwards if \( \rho_b > \rho_l \), upwards if \( \rho_b < \rho_l \)), \( g \) is the gravitational acceleration (m/s\(^2\)), \( \rho_b \) is the mass density of the ball (kg/m\(^3\)), \( \rho_l \) is the mass density of the liquid (kg/m\(^3\)) and \( \mu \) is the dynamic viscosity (kg/m*s)[14].

As the velocity of a ball through the liquid increases, flow past the ball no longer remains perfectly laminar. The formation of eddies is observed and turbulence groups in Turbulent movement is very difficult to analyze, and one has to resort to experimental methods in order to treat this situation. The degree of turbulence is usually measure by the magnitude of a dimensionless number \( Re \) known as the Reynold's Number. The Reynold's Number for a ball is given by:[14,15]

\[
Re = \frac{2rv}{\mu} \rho_f
\]  

(10)

It is found experimentally that turbulence is exists and important whenever \( Re \geq 1 \).

IV. The practical work

The block diagram in figure(4) show the suggested processing steps for determine the model equations for falling ball in the water, in which the work was divided into several phases, during each phase, explain the practical work and detail of the uses, tools, devices, algorithms and software.

### Used inside the water camera to capture a video clip for falling balls in basin water .then extract still Image from these clip

Using Ulead video studio software.

### Built MATLAB program to calculate change of displacement(\( \Delta r \)),velocity (v),and Acceleration(A) as a function of motion time for each frame image .

### Using Table Curve software (TC) to Estimate the functions (\( \Delta r \)), (v), and (A) as a function of motion Time (t) and determine a mathematical model for fitting functions to find The relation between Distance (D) distance between camera and falling ball and the model equations .

### Verification: Compared the theoretical and practical data

**Figure (4) : The Block diagram explain the steps of estimate the motion parameters for falling ball in the water.**
A. Extract Still Images From Video Clip

Using digital waterproof camera (SONY DSC-TX10, Resolution 16.20 megapixels) inside the water. Basin dimensions is (80, 37, 50) cm$^3$ filled with water as show in figure (5a), have been used two kind of ball (Billiard and snooker) have different scale tabulated its information in table (1). Falling ball vertically at height 8 cm from the basin water surface, for every 10 cm at difference Distances (D) between the camera and falling ball, started from 30, 40, 50, 60, 70 and 80 cm, have been taken video clip. Using Ulead 2011 software to convert video clip for falling ball in water into still images (frames) this frames save in JPEG format, these video clip for each second converted into (30 frame per sec(fps)) first frame $f(x_1,y_1)$ started from initial time=0 sec, second frame $f(x_2,y_2)$ its time 1/30 sec, third frame $f(x_3,y_3)$ its time 2/30 sec, ...etc. represent these images extract as a frame of time $f(x_t,y_t)$. Thus by determining the difference between two points from, $[f(x_{t+1},y_{t+1})-f(x_t,y_t)]$ as shown in the figure(5b), to reach the bottom of the basin.

![Figure 5 (a) the used basin: (1) basin water, (2) fall ball, (3) waterproof camera(sony), (4) source of light. (b) The sketch of the falling ball in basin.](image)

Table (1) show some information about the Two type of balls (Billiard and snooker).

<table>
<thead>
<tr>
<th></th>
<th>snooker ball</th>
<th>billiard ball</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (V)</td>
<td>77.5 cm$^3$</td>
<td>37 cm$^3$</td>
</tr>
<tr>
<td>Diameter</td>
<td>5.29 cm</td>
<td>4.14 cm</td>
</tr>
<tr>
<td>Weight (m)</td>
<td>150 g</td>
<td>60 g</td>
</tr>
<tr>
<td>Density = $\frac{m}{V}$</td>
<td>1.93 (g/cm$^3$)</td>
<td>1.62 (g/cm$^3$)</td>
</tr>
</tbody>
</table>

B. Analysis Algorithms:

MATLAB software using the suggested algorithms which has been build. The first algorithm to compute the scale factor (Scf), which this is performed by determining the ball diameter($d_{\text{pixel}}$) in picture plane by selecting two points on the boundary in the image using computer mouse, see figure (6). In this work several frames images compute scale factor for each frame by using the flowing Algorithm, this scale factor will be used later for other computations.

![Figure 6. shows the Dotted line in picture plane to determine the ball diameter.](image)
The second Algorithm describe the process in detail how can be calculate the falling ball location \( r(x, y) \), change of displacement \( \Delta r \), the Velocity \((v)\) and the Acceleration \( (A)\) for each images frame, by using the following algorithm

**Algorithm (1) Determine of Scale factor\((Scf)\) for the frame image**

**Input:**
1- The image of falling ball in the basin \((img)\).
2- Real Diameter of The ball \( D_b\) (cm).

**Output:**
Scale factor \((Scf)\).

**Start algorithm**
1- Load image\((img)\), to determine diameter of the ball \((x, y)\).
2- Manually selected upper and lower ball boundary points to determine the Diameter (corresponding points) of Ball’s in the image plane to compute ball diameter in pixel.
   - This process can be explain in two step:
     i) Used the property (mouse click) and determine the upper point boundaries \((x_1, y_1)\) of ball image.
     ii) Move and click mouse, Used the property again (mouse click) and determine the lower point boundaries \((x_2, y_2)\) of ball image, see the figure (6).
3- Compute the Diameter length in pixels between point \((1)\) \((x_1, y_1)\) and point \((2)\) \((x_2, y_2)\) using:
   \[
   d_{\text{pixel}} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
   \]
4- Compute scaling factor for this image frame using:
   \[
   Scf = \frac{D_{b\text{image}}}{d_{\text{pixel}}}
   \]
5- Print Scf.
6- End algorithm.

The second Algorithm describe the process in detail how can be calculate the falling ball location \( r(x, y) \), change of displacement \( \Delta r \), the Velocity \((v)\), and the Acceleration \( (A)\) for each images frame, by using the following algorithm.
Algorithm (2) calculate the location \( r(x,y) \), change of displacement(\( \Delta r \)), the Velocity(\( v \)) and the Acceleration(\( A \)) of the moving ball inside the water.

Input:
1. Scale factor (Scf).
2. The sequence n-frames (\( img_{(1)} \),\( img_{(2)} \),…..\( img_{(n)} \)).

Output:

i. The distance between two balls images both in pixels and centimeters, (\( \Delta r \))pixel, \( \Delta r \)(cm).
ii. The velocity\( (v) \) in centimeters.
iii. The Acceleration\( (A) \) in centimeters.

Step 1 : manually selected two bottom points in input images (\( img_{(i)} \)) as a fallow:

i. Used the property of (mouse click) and determine the bottom point \( (x_i,y_i) \) of ball image (\( img_{(i)} \)).
ii. Used the property again (mouse click) and determine the bottom point \( (x_{i+1},y_{i+1}) \) of ball image (\( img_{(i+1)} \)).

Step 2 : Compute the distance in pixels between the two points \( (x_i,y_i), (x_{i+1},y_{i+1}) \) using:

\[
\Delta r_{\text{pixel}} = \sqrt{ (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 }
\]

Step 3 : Converted \( \Delta r \) from pixels to centimeters (cm) using:

\[
\Delta r_{\text{cm}} = Scf \times \Delta r_{\text{pixel}}
\]

Step 4 : Compute the velocity\( (v_i) \) in centimeter, depending on \( \Delta r_{\text{cm}} \) using the following equation:

\[
v_i = \frac{\Delta r}{\Delta t}
\]

Where \( t = \frac{1}{n} \) sec, and \( \Delta t \), represent the difference of two sequence frame.

Step 5 : input image (\( img_{(i+2)} \)) and repeat steps (from step 1 to step 4) for (\( img_{(i+1)} \)) and (\( img_{(i+2)} \))sequence frame by frame to compute \( (v_{i+1}) \).

Step 6 : Compute \( \Delta v_{\text{cm}} \) using :

\[
\Delta v_{\text{cm}} = v_{i+1} - v_i
\]

Step 7 : Compute the Acceleration (\( A \)) in centimeter as following equation :

\[
A = \frac{\Delta v}{\Delta t}
\]

Step 8 : End algorithm.
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C. Best Models Estimation:

Used “Table Curve 2D Version 5.01” to fitting the practical $r$, $\nu$ and A data for different Distance(D) between camera and falling ball to introduce appropriate mathematical functions for computed $\Delta r$, $\nu$ and A values for the captured images. The estimated curves for the motion parameters $r$, $\nu$ and A are shown in figure(7) for distance (D=50 cm) for two balls. The other fitting $r$, $\nu$ and A data at Distance D=70,80 tabulates in table(2).

Figure (7): shows fitting curve functions for the time: (a) $\Delta r$ (b) Velocity ($\nu$) and (c)Acceleration(A) functions for billiard ball on the left side, and snooker ball on the right side, at Distance (D=50cm)
Table (2) Shows, functions values, corresponding to the time for each two types of balls at distance D=70,80 cm

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>(D=70) cm</th>
<th>(D=80) cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Change of displacement ((\Delta r))</td>
<td>Velocity((v))</td>
</tr>
<tr>
<td></td>
<td>Billiard Ball</td>
<td>Snooker Ball</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>5.57</td>
<td>4.47</td>
</tr>
<tr>
<td>3</td>
<td>4.48</td>
<td>5.02</td>
</tr>
<tr>
<td>4</td>
<td>3.8</td>
<td>4.88</td>
</tr>
<tr>
<td>5</td>
<td>2.78</td>
<td>3.78</td>
</tr>
<tr>
<td>6</td>
<td>2.67</td>
<td>3.15</td>
</tr>
<tr>
<td>7</td>
<td>1.99</td>
<td>3.53</td>
</tr>
<tr>
<td>8</td>
<td>2.72</td>
<td>3.24</td>
</tr>
<tr>
<td>9</td>
<td>2.25</td>
<td>3.08</td>
</tr>
</tbody>
</table>

The estimate best equations for the relationship between the three motion parameters change of displacement \((\Delta r)\), Velocity \((v)\), and Acceleration \((A)\) as a function with the time as follows:

\[
\Delta r = a_i + \frac{b_i}{t^2} + c_i \tag{11}
\]

\[
V = a_j + \frac{b_j}{t^2} + c_j \ln(t) \tag{12}
\]

\[
A = a_k + \frac{b_k}{t^2} + c_k e^{-t} \tag{13}
\]

Where the values of the constants \(a_{(i,j,k)}\), \(b_{(i,j,k)}\), and \(c_{(i,j,k)}\) and correlation \((\rho^2)\) are reported in Table (3) and (4) for each balls with different Distances \((D)\).

Table (3): The parameters of the fitting equations for the three functions \((\Delta r,v\text{ and }A)\) and real different Distance \((D)\) for Billiard Ball.

<table>
<thead>
<tr>
<th>Distance ((D)) cm</th>
<th>Change of displacement ((\Delta r))</th>
<th>Velocity((v))</th>
<th>Acceleration ((A))</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.84</td>
<td>-23.16</td>
<td>-21.35</td>
</tr>
<tr>
<td>55</td>
<td>0.994</td>
<td>-16.86</td>
<td>-16.92</td>
</tr>
<tr>
<td>70</td>
<td>-0.202</td>
<td>-21.25</td>
<td>-21.02</td>
</tr>
<tr>
<td>75</td>
<td>0.479</td>
<td>19.67</td>
<td>-20.12</td>
</tr>
<tr>
<td>80</td>
<td>0.813</td>
<td>11.90</td>
<td>-12.7</td>
</tr>
</tbody>
</table>

\[
\rho^2 = 0.85\tag{11}
\]

Table (4): The parameters of the fitting equations for three functions \((\Delta r,v\text{ and }A)\) and real different Distance \((D)\) for Snooker Ball.

<table>
<thead>
<tr>
<th>Distance</th>
<th>Change of displacement ((\Delta r))</th>
<th>Velocity((v))</th>
<th>Acceleration ((A))</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.84</td>
<td>-23.16</td>
<td>-21.35</td>
</tr>
<tr>
<td>55</td>
<td>0.994</td>
<td>-16.86</td>
<td>-16.92</td>
</tr>
<tr>
<td>70</td>
<td>-0.202</td>
<td>-21.25</td>
<td>-21.02</td>
</tr>
<tr>
<td>75</td>
<td>0.479</td>
<td>19.67</td>
<td>-20.12</td>
</tr>
<tr>
<td>80</td>
<td>0.813</td>
<td>11.90</td>
<td>-12.7</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>(D) cm</th>
<th>a_i</th>
<th>b_i</th>
<th>c_i</th>
<th>(\rho^*)</th>
<th>a_j</th>
<th>b_j</th>
<th>c_j</th>
<th>(\rho^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.416</td>
<td>11.33</td>
<td>-12.78</td>
<td>0.95</td>
<td>-2.787</td>
<td>206.1</td>
<td>0.97</td>
<td>55.45</td>
</tr>
<tr>
<td>55</td>
<td>-0.55</td>
<td>24.2</td>
<td>-23.35</td>
<td>0.99</td>
<td>0.8333</td>
<td>115.4</td>
<td>0.94</td>
<td>33.75</td>
</tr>
<tr>
<td>70</td>
<td>1.632</td>
<td>14.71</td>
<td>-16.39</td>
<td>0.95</td>
<td>-3.849</td>
<td>250.3</td>
<td>0.94</td>
<td>55.88</td>
</tr>
<tr>
<td>75</td>
<td>-1.32</td>
<td>31.5</td>
<td>-30.75</td>
<td>0.97</td>
<td>-2.944</td>
<td>212.5</td>
<td>0.92</td>
<td>57.3</td>
</tr>
<tr>
<td>80</td>
<td>1.590</td>
<td>11.76</td>
<td>-13.36</td>
<td>0.87</td>
<td>-3.258</td>
<td>217.5</td>
<td>0.92</td>
<td>58.200</td>
</tr>
</tbody>
</table>

The relation between \((a, b, c)\)-parameters as a function of the Distance \((D)\), using table curve software obtain the following equations, for first ball (billiard):

\[
\begin{align*}
a_i &= -1.01 + \frac{8283.67}{D^2} - 2.15 \times 10^{22} e^{-D} \\
b_i &= 17.98 - 1.09 \times 10^{-34} e^D + 2.7 \times 10^{22} e^{-D} \\
c_i &= -18.7 + 1.08 \times 10^{-34} e^D - 1.4 \times 10^{22} e^{-D}
\end{align*}
\]

(14)

For second ball (snooker):

\[
\begin{align*}
a_i &= 1.6 - 5.4 \times 10^{-37} e^D - 1.1 \times 10^{21} e^{-D} \\
b_i &= 14.7 - 5.3 \times 10^{-35} e^D - 1.75 \times 10^{22} e^{-D} \\
c_i &= -16.4 + 5.4 \times 10^{-35} e^D + 1.9 \times 10^{22} e^{-D}
\end{align*}
\]

(15)

Here the eqs.(14) and (15) representing the relation between \((a, b, c)\)-parameters and Distance \((D)\) for billiard and snooker balls. By substituting these equations into eq. (11) get mathematical models that relates change displacement \((\Delta r)\), Time \((t)\) and Distance \((D)\), for first ball (billiard):

\[
\Delta r = -1.01 + \frac{8283.67}{D^2} - 2.15 \times 10^{22} e^{-D} + \frac{(17.98 - 1.09 \times 10^{-34} e^D + 2.7 \times 10^{22} e^{-D})}{t} + \frac{(-18.7 + 1.08 \times 10^{-34} e^D - 1.4 \times 10^{22} e^{-D})}{t^2}
\]

(16)

For second ball (snooker):

\[
\Delta r = (1.6 - 5.4 \times 10^{-37} e^D - 1.1 \times 10^{21} e^{-D}) + \frac{(14.7 - 5.3 \times 10^{-35} e^D - 1.75 \times 10^{22} e^{-D})}{t} + \frac{(-16.4 + 5.4 \times 10^{-35} e^D + 1.9 \times 10^{22} e^{-D})}{t^2}
\]

Here eqs.(16) and (17) represented the mathematical models for \(\Delta r\) parameter of motion billiard and snooker falling ball in water. While the mathematical model for velocity \((v)\) parameter can be estimated in same way of estimated \(\Delta r\), the estimated parameter are as follow, for first ball (billiard):

\[
\begin{align*}
a_j &= -15.5 + 0.11 D + \frac{2354.4}{D^{1.5}} \\
b_j &= 955.2 - 6.86 D - \frac{151564.3}{D^{1.5}}
\end{align*}
\]

(18)

For second ball (snooker):

\[
\begin{align*}
a_j &= -7.34 + \frac{24618.8}{D^2} - 2.7 \times 10^{22} e^{-D} \\
b_j &= 327.02 - \frac{646806.5}{D^2} + 7.1 \times 10^{23} e^{-D}
\end{align*}
\]

(19)

Where the eqs.(18) and (19) representing the relation between \((a, b)\)-parameters and Distance \((D)\), for billiard ball. By substituting these equations into eq.(12) get mathematical models that relates velocity \((v)\), Time \((t)\) and Distance \((D)\), for first ball (billiard):

\[
v = (-15.5 + 0.11 D + \frac{2354.4}{D^{1.5}}) + \left(955.2 - 6.86 D - \frac{151564.3}{D^{1.5}}\right) \ln\left(\frac{t}{t^2}\right)
\]

(20)
For second ball(snooker):

\[ v = (-7.34 + \frac{24618.4}{d^2} - 2.7 \times 10^{22}e^{-D}) + \left(327.02 - \frac{646806.5}{D^2} + 7.1 \times 10^{23}e^{-D} \right) \frac{\ln(t)}{t^2} \]  \hspace{1cm} (21)

Here eqs.(20) and (21) represent the mathematical models for \( v \) parameter of motion of billiard and snooker ball velocity of the falling in water.

Also can be found mathematical model for Acceleration (A)- parameters with the same way estimated in of \( \Delta r \) and \( v \), the estimated parameter are as follows, for first ball(billiard):

\[
\begin{align*}
    a_{k1} &= 57.8 - 7.8 \times 10^{-5}D^3 + 5.8 \times 10^{-34}e^D \\
    b_{k1} &= -1954.1 + 0.002D^3 - 3.2 \times 10^{-32}e^D \\
    c_{k1} &= 5099.1 - 0.005D^3 + 8.4 \times 10^{-32}e^D
\end{align*}
\]  \hspace{1cm} (22)

For second ball(snooker):

\[
\begin{align*}
    a_{k2} &= 83.9 - \frac{153898.9}{D^2} + 1.7 \times 10^{25}e^{-D} \\
    b_{k2} &= -5374.7 + \frac{14256534}{D^2} - 1.5 \times 10^{25}e^{-D} \\
    c_{k2} &= 14437.3 - \frac{38700532}{D^2} + 4.1 \times 10^{25}e^{-D}
\end{align*}
\]  \hspace{1cm} (23)

Where the eqs.(22) and (23) representing the relation between \( (a_k, b_k \) and \( c_k \)-parameters and Distance (D), for an acceleration of the falling billiard and snooker ball respectively. By substituting these equations into (13) get mathematical models that relates Acceleration (A), Time (t) and Distance (D) as follow, for the first ball(billiard):

\[
A = (57.8 - 7.8 \times 10^{-5}D^3 + 5.8 \times 10^{-34}e^D) + \left(\frac{-1954.1 + 0.002D^3 - 3.2 \times 10^{-32}e^D}{t^2}\right) + (5099.1 - 0.005D^3 + 8.4 \times 10^{-32}e^D)e^{-t}
\]  \hspace{1cm} (24)

For the second ball(snooker):

\[
A = (83.9 - \frac{153898.9}{D^2} + 1.7 \times 10^{25}e^{-D}) + \left(\frac{-5374.7 + \frac{14256534}{D^2} - 1.5 \times 10^{25}e^{-D}}{t^2}\right) + (14437.3 - \frac{38700532}{D^2} + 4.1 \times 10^{25}e^{-D})e^{-t}
\]  \hspace{1cm} (25)

Here eqs.(24) and (25) represent the mathematical model for the acceleration(A) of ball billiard and snooker motion in water.

D. Models Verification:

The verification have been performed by canceled the practical results for distance (D=60 cm) from fitting operation in previous modeling process. In order to determine their values theoretically from the estimated mathematical model and make a comparison between the experimental and theoretical data. Now will be reviewing the graphically details for the practical and theoretical for each functions of billiard and snooker falling ball, see figure(8,9).
Mathematical Model Estimation for Falling Ball in Water Based on the Capture Images Using SONY Camera

Figure (8) Matching the practical graphic at left side and theoretical graphic at the right side (a) of $\Delta r$, (b) Velocity($v$), and (c) Acceleration($A$) for the falling billiard ball.
Figure (9) Matching the practical graphic at left side and theoretical graphic at the right side (a) of Δr, (b) Velocity(υ), and (c) Acceleration(A) for the falling for snooker ball.

As in figure (8) and (9) notes that the excellent match between the practical and theoretical curves. An error percentage resulting between the practical and theoretical for Distance (D=60 cm) tabulated in the table (5).

Table (5) Shows the error percentage resulting between the practical and theoretical data, D=60 cm

<table>
<thead>
<tr>
<th>Function</th>
<th>Sony camera</th>
<th>Billiard ball</th>
<th>Error percentage %</th>
<th>Snooker ball</th>
<th>Error percentage %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta (cm) r</td>
<td>5.6</td>
<td>4.96</td>
<td>11.4%</td>
<td>4.95</td>
<td>4.9</td>
</tr>
</tbody>
</table>
Mathematical Model Estimation for Falling Ball in Water Based on the Capture Images Using SONY Camera

<table>
<thead>
<tr>
<th>velocity</th>
<th>Accelera-</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.15</td>
<td>362.5</td>
</tr>
<tr>
<td>35.1</td>
<td>266.25</td>
</tr>
<tr>
<td>0.142%</td>
<td>26.5</td>
</tr>
<tr>
<td>26.25</td>
<td>255</td>
</tr>
<tr>
<td>25.749</td>
<td>250</td>
</tr>
<tr>
<td>1.90%</td>
<td>1.96%</td>
</tr>
</tbody>
</table>

V. Conclusions

From the previous results of the present work, a new three mathematical models for $\Delta r$, $v$ and $A$ for the objects in images captured by (SONY DSC-TX10, Resolution 16.20 megapixels) camera;

- First model: The change in displacement ($\Delta r$) as in Eqs.(16) and (17), its accuracy 4.96% for Billiard ball and 1.01% for snooker ball.
- Second model: The velocity ($v$) as in Eqs.(20) and (21), its accuracy 0.142% for Billiard ball and 1.9% for snooker ball.
- Third model: The Acceleration ($A$) as in Eqs.(24) and (25) its accuracy 26.5% for Billiard ball and 1.96% for snooker ball.

By Increasing the surface area for the ball the buoyancy force of the fluid increases to the highest (buoyancy) and vice versa. as well as increase of the mass cause increase in the falling speed of the ball, where can be note that the speed of balls (to the biggest mass, higher density, larger size) has the largest speed compared mass least. In billiard ball least mass density and size result in a zigzag movement during the falling , cause the error rate.

REFERENCES

[14] "Fluid Mechanics", Scott A. Shearer, Professor Jeremy R. Hudson,Measurement Laboratory, Investigation No. 3,Graduate Teaching Assistant,Biosystems and Agricultural Engineering.